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Heat and mass transfer across tall cavities filled with gas–vapour mixtures: the fully developed regime

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Abstract—Closed form expressions for the fully developed velocity, temperature and concentration profiles in a vertical channel are found by solving the equations for a cavity in the limit as the aspect ratio tends to infinity. We consider plane, steady, laminar, Boussinesq flow of an ideal gas–vapour mixture. The vertical walls are held at different constant temperatures and compositions, are impermeable to the gas and non-slip. The finite mass transfer effects of interfacial velocity and interdiffusion of enthalpy are included.

1. INTRODUCTION

In 1954, Batchelor [1] investigated heat transfer across cavities filled with a single fluid. He realised that if the aspect ratio is large enough, a fully developed regime could exist in the region sufficiently far from either of the horizontal surfaces. Here the temperature would vary linearly across the cavity while the purely vertical velocity would have a symmetric cubic profile. This is identical to the solution for a cavity bounded only by two vertical walls, reported in 1946 by Jones and Furry [2] and in many textbooks [3–6]. Various attempts [1, 7–9] to find the minimum aspect ratio for which this regime exists (in the single fluid case), will be treated in a subsequent paper.

In 1972, Aung [10] found exact solutions for fully developed temperature and velocity profiles in vertical channels open at the top and bottom to a uniform environment. The temperature variation is unchanged, but the velocity profile contains an additional constant governing the influence of the ambient temperature and hence the net vertical mass flux and the deviation of the velocity profile from odd symmetry. Aung also solved the problem for constant wall heat fluxes.

While the most familiar fully developed flows (e.g. plane and axisymmetric Poiseuille flow, and that of Jones and Furry [2]) are unidirectional, exact solutions have been published for the plane flow of a pure fluid between vertical [11] and horizontal [12] parallel plates with uniform normal interfacial velocity and buoyancy effects due to heat transfer.

In the first treatment of the effect of a diffusing species, Nelson and Wood [13] showed that Aung's results for the constant wall temperature case may be simply extended to include constant wall vapour mass fraction provided that the vapour flux is small enough for the interfacial velocity and the interdiffusion of enthalpy to be neglected.

In the study by T. S. Lee *et al.* of heat and mass transfer across finite vertical channels [14], the high mass transfer rate effects of interfacial velocity and the interdiffusion of enthalpy are included but the mass transfer boundary conditions (constant and zero flux at the two opposing vertical walls) prevent the establishment of a fully developed regime, although at the lower mass transfer rates the results indicate invariant axial velocity profiles over much of the channel height if one or other of the buoyancy forces is dominant (i.e. very large or very small $|N|$). It is shown later in the present paper that a fully developed regime can exist with non-zero transverse velocity, but only if it is constant.

Recent numerical work on heat and mass transfer in vertical cavities [15] and rectilinear [16, 17] and axisymmetric [18–20] conduits filled with gas–vapour mixtures has suggested that mass transfer can significantly change both the flow field and the overall energy transfer rate—a conclusion supported by the present analysis.

2. GENERAL MATHEMATICAL MODEL

In this section, we present the dimensionless field equations, boundary conditions and wall fluxes applicable for gas–vapour mixtures in vertical, two-dimensional cavities of arbitrary aspect ratio (the geometry and boundary conditions are shown in Fig. 1). In the following section, we derive their limiting forms for large aspect ratio. The field variables are non-dimensionalised so as to ensure that their values are

NOMENCLATURE

$\mathscr{A}$	cavity aspect ratio
B	blowing number, $(\omega_{A_l}-\omega_{A_0})/(\omega_{A_0}-1)$
$c_1,\ldots,c_4$	constants
c_p	isobaric specific heat
D_{AB}	binary diffusivity
$\mathbf{e}$	energy flux relative to fixed coordinates
g	gravitational field strength
Gr^*	modified Grashof number, $g\beta(T_l-T_0)(1+N)l^3/\nu^2$
h	specific enthalpy
H	reduced vapour enthalpy base, $\rho h_{A_0}D_{AB}/(T_l-T_0)k$
k	thermal conductivity
l	cavity width
Le_A	vapour Lewis number, $k/\rho c_{p_A}D_{AB}$
Le_{AB}	interdiffusion Lewis number, $k/\rho(c_{p_A}-c_{p_B})(\omega_{A_0}-1)D_{AB}$
$\mathbf{n}$	mass flux relative to fixed coordinates
N	buoyancy ratio, $\zeta(\omega_{A_l}-\omega_{A_0})/\beta(T_l-T_0)$
Nu^*	modified Nusselt number, $-e_xl(T_l-T_0)k$, at $X=0, 1$
p	(total) pressure
P	reduced pressure, $l^2(p+\rho gy)/\nu^2Gr^*\mathscr{A}\rho$
Sc	Schmidt number, ν/D_{AB}
T	temperature
u, v	horizontal and vertical components of $\mathbf{v}$
U	reduced horizontal velocity component, $lu/D_{AB}\ln(B+1)$
$\mathbf{v}$	primitive velocity vector
V	reduced vertical velocity component, $lv/\nu Gr^*$
x, y	primitive horizontal and vertical coordinates
X	reduced horizontal coordinate, $X=x/l$
Y	reduced vertical coordinate, $Y=y/\mathscr{A}l$.

Greek symbols

β	thermal coefficient of volumetric expansion
ζ	vapour mass fraction coefficient of volumetric expansion
θ	reduced temperature, $(T-T_0)/(T_l-T_0)$
ν	kinematic viscosity
ρ	density
σ	function defined by equation (41)
ϕ	reduced mass fraction, $(\omega_A-\omega_{A_0})/(\omega_{A_l}-\omega_{A_0})$
ω	mass fraction.

Subscripts

0	at the vertical wall $x=0$
1	at the vertical wall $X=1$
A	species A, the vapour
B	species B, the gas
l	at the vertical wall $x=l$
x	horizontal component
y	vertical component
∞	in the limit $\mathscr{A}\to\infty$.

of order unity in the tall cavity limit, as will be apparent once the solution is obtained. The independent variables are normalised by transforming the cavity to a square domain.

For arbitrary aspect ratio, the problem depends on eight dimensionless parameters, $\mathscr{A}$, B, N, Sc, Le_A, Le_{AB}, Gr^* and H. The last of these, H, the dimensionless base enthalpy of the vapour, does not enter into the cavity equations at all but it does occur in the energy flux at the vertical walls.

The interdiffusion Lewis number, Le_{AB}, controls the advection of energy apart from that which may be accounted for by the specific heat of the vapour. Some of this is because the mixture specific heat differs from that of the vapour, and some is due to the species interdiffusion effect. The ratio of the mass transfer driving force, B, to Le_{AB} governs the importance of the interdiffusive energy flux relative to conduction. For very dilute vapours in cavities of arbitrary aspect ratio, it may be more convenient to use a Lewis number based on the gas specific heat, but the present choice leads to the simplest formulation for the fully developed regime.

2.1. *The field equations*

Assuming that the density variation is only important in the buoyancy force term, which may be modelled as a linear function of temperature and mass fraction, and taking the y-direction as vertical, the equations of continuity and motion for the mixture are [21]:

$$\frac{\partial U}{\partial X}+\frac{Sc\,Gr^*}{\mathscr{A}\ln(B+1)}\frac{\partial V}{\partial Y}=0 \tag{1}$$

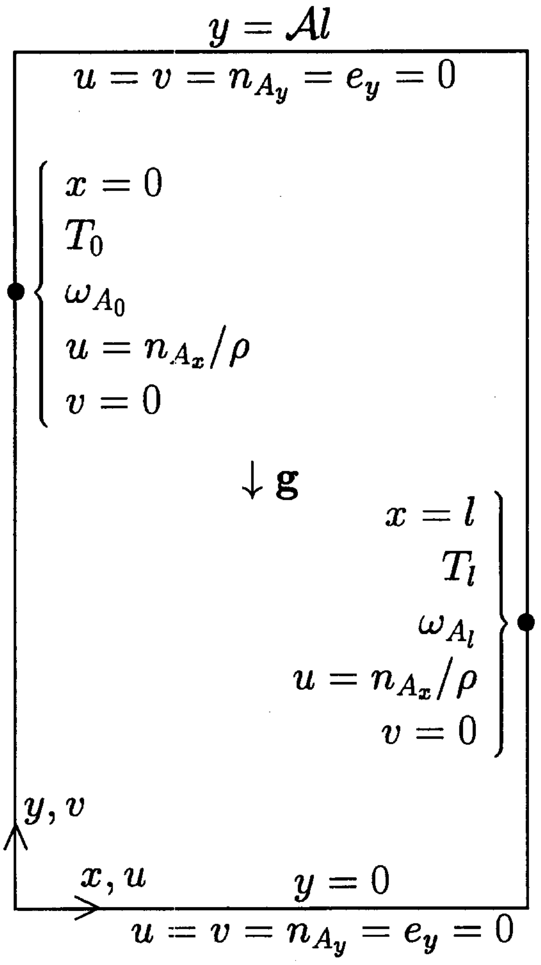

Fig. 1. Geometry and boundary conditions for cavity of arbitrary aspect ratio.

$$\frac{\ln^2(B+1)}{Gr^*\,Sc^2\mathscr{A}}U\frac{\partial U}{\partial X}+\frac{\ln(B+1)}{Sc\,\mathscr{A}^2}V\frac{\partial U}{\partial Y}$$

$$=-\frac{\partial P}{\partial X}+\frac{\ln(B+1)}{Gr^*\,Sc\,\mathscr{A}}\left(\frac{\partial^2 U}{\partial X^2}+\frac{1}{\mathscr{A}^2}\frac{\partial^2 U}{\partial Y^2}\right) \quad (2)$$

$$\frac{\ln(B+1)}{Sc}U\frac{\partial V}{\partial X}+\frac{Gr^*}{\mathscr{A}}V\frac{\partial V}{\partial Y}$$

$$=-\frac{\partial P}{\partial Y}+\frac{\theta+N\phi}{1+N}+\frac{\partial^2 V}{\partial X^2}+\frac{1}{\mathscr{A}^2}\frac{\partial^2 V}{\partial Y^2} \quad (3)$$

With the temperature and mass fraction varying through the cavity, supersaturation of the vapour is possible. Whether or not this leads to condensation in the body of the cavity, and, if so, at what rate droplet formation and growth proceed, depends on the presence, nature and distribution of nucleating aerosols. An analysis of these molecular kinetics lies outside the scope of continuum fluid mechanics and this project. The problem is investigated in ref. [15], and the physics is described in meteorological treatises [22, 23]. If condensation takes place only at the boundaries, the conservation of vapour in the body of the cavity requires [24]:

$$\nabla\cdot\mathbf{n}_A=0 \quad (4)$$

where, for Fickian diffusion,

$$\mathbf{n}_A=\rho(\omega_A\mathbf{v}-D_{AB}\nabla\omega_A) \quad (5)$$

The species equation, with constant diffusivity, is therefore

$$\ln(B+1)U\frac{\partial\phi}{\partial X}+\frac{Gr^*\,Sc}{\mathscr{A}}V\frac{\partial\phi}{\partial Y}=\frac{\partial^2\phi}{\partial X^2}+\frac{1}{\mathscr{A}^2}\frac{\partial^2\phi}{\partial Y^2} \quad (6)$$

Neglecting gravitational potential energy, the equation of energy is [24],

$$\nabla\cdot\mathbf{e}=0 \quad (7)$$

and if species A and B form a perfect gaseous mixture (partial specific enthalpies independent of composition and species specific heats independent of temperature [25]) and emission and absorption of radiation, viscous dissipation, advection of kinetic energy, and the Dufour effect may be neglected, the energy flux is [26],

$$\mathbf{e}=-k\nabla T+\rho\left\{\left[(c_{p_A}-c_{p_B})\left(\frac{\mathbf{n}_A}{\rho}-\mathbf{v}\right)+c_{p_A}\mathbf{v}\right](T-T_0)\right.$$

$$\left.+(h_{A_0}-h_{B_0})\left(\frac{\mathbf{n}_A}{\rho}-\mathbf{v}\right)+h_{A_0}\mathbf{v}\right\} \quad (8)$$

It does not make sense to treat the mixture specific heat as a constant if the interdiffusion term is included, as this introduces a term into the energy flux divergence which is dependent on the temperature datum for enthalpy, which must be arbitrary. If the mixture specific heat is treated as constant in the energy flux divergence (the energy equation), a spurious source term occurs.

For constant thermal conductivity, the energy equation is therefore:

$$\frac{\partial^2\theta}{\partial X^2}+\frac{1}{\mathscr{A}^2}\frac{\partial^2\theta}{\partial Y^2}$$

$$=\left\{\frac{1}{Le_{AB}}\left[(B\phi+1)\ln(B+1)U-B\frac{\partial\phi}{\partial X}\right]\right.$$

$$\left.+\frac{\ln(B+1)}{Le_A}U\right\}\frac{\partial\theta}{\partial X}$$

$$+\left\{\frac{1}{Le_{AB}}\left[(B\phi+1)Gr^*\,Sc\,V-\frac{B}{\mathscr{A}}\frac{\partial\phi}{\partial Y}\right]\right.$$

$$\left.+\frac{Gr^*\,Sc}{Le_A}V\right\}\frac{1}{\mathscr{A}}\frac{\partial\theta}{\partial Y} \quad (9)$$

2.2. *Boundary conditions*

Take the vertical walls to be at uniform temperatures and vapour mass fractions, and permeable to the vapour only. Take the horizontal walls to be adiabatic and impermeable, and all walls to be non-slip. The dimensionless boundary conditions are thus:

$$U(0,Y) = \frac{B}{\ln(B+1)} \left.\frac{\partial\phi(0,Y)}{\partial X}\right|_{X=0} \quad (10)$$

$$U(1,Y) = \frac{B}{(B+1)\ln(B+1)} \left.\frac{\partial\phi(1,Y)}{\partial X}\right|_{X=1} \quad (11)$$

$$U(X,0) = U(X,1) = 0 \quad (12)$$

$$V(0,Y) = V(1,Y) = 0 \quad (13)$$

$$V(X,0) = V(X,1) = 0 \quad (14)$$

$$\theta(0,Y) = \theta(1,Y) - 1 = 0 \quad (15)$$

$$\phi(0,Y) = \phi(1,Y) - 1 = 0 \quad (16)$$

$$\frac{1}{\mathscr{A}}\frac{\partial\theta}{\partial Y} = \frac{1}{\mathscr{A}}\frac{\partial\phi}{\partial Y} = 0, \quad \text{at } Y = 0, 1 \quad (17)$$

2.3. *The wall fluxes*

The horizontal component of vapour mass flux relative to stationary coordinates, from equation (5), may be simplified at the vertical walls by the boundary conditions on U, equations (10)–(11), to give

$$n_{A_x} = \rho u, \quad x = 0, l \quad (18)$$

thus the dimensionless horizontal component of velocity, U, may be used as a dimensionless vapour flux. A Sherwood number, i.e. a mass flux normalised by the difference in vapour mass fraction across the cavity is less useful as explained in detail by Spalding [27, 28].

The dimensionless energy fluxes at the walls are:

$$Nu_0^* = \left.\frac{\partial\theta}{\partial X}\right|_{X=0} - H\ln(B+1)U(0,Y) \quad (19)$$

$$Nu_1^* = \left.\frac{\partial\theta}{\partial X}\right|_{X=1} - \left(H + \frac{1}{Le_A}\right)\ln(B+1)U(1,Y) \quad (20)$$

3. THE TALL CAVITY LIMIT

We seek an asymptotic solution of this problem for large aspect ratio.

3.1. *Equation set for first approximation*

In the limit $\mathscr{A} \to \infty$, equations (1)–(3), (6), (9) and (10)–(17) become

$$\frac{\partial U_\infty}{\partial X} = 0 \quad (21)$$

$$\frac{\partial P_\infty}{\partial X} = 0 \quad (22)$$

$$\frac{\partial^2 V_\infty}{\partial X^2} - \frac{\ln(B+1)}{Sc}U_\infty\frac{\partial V_\infty}{\partial X} - \frac{\partial P_\infty}{\partial Y} + \frac{\theta_\infty + N\phi_\infty}{1+N} = 0 \quad (23)$$

$$\frac{\partial^2\phi_\infty}{\partial X^2} - \ln(B+1)U_\infty\frac{\partial\phi_\infty}{\partial X} = 0 \quad (24)$$

$$\frac{\partial^2\theta_\infty}{\partial X^2} - \left\{\frac{1}{Le_{AB}}\left[(B\phi_\infty + 1)\ln(B+1)U_\infty - B\frac{\partial\phi_\infty}{\partial X}\right] + \frac{\ln(B+1)}{Le_A}U_\infty\right\}\frac{\partial\theta_\infty}{\partial X} = 0 \quad (25)$$

subject to:

$$\theta_\infty(0,Y) = \phi_\infty(0,Y) = 0 \quad (26)$$

$$\theta_\infty(1,Y) = \phi_\infty(1,Y) = 1 \quad (27)$$

$$U_\infty(0,Y) = \frac{B}{\ln(B+1)}\left.\frac{\partial\phi_\infty}{\partial X}\right|_{X=0} \quad (28)$$

$$U_\infty(1,Y) = \frac{B}{(B+1)\ln(B+1)}\left.\frac{\partial\phi_\infty}{\partial X}\right|_{X=1} \quad (29)$$

$$V_\infty(0,Y) = V_\infty(1,Y) = 0 \quad (30)$$

$$U_\infty(X,0) = U_\infty(X,1) = 0 \quad (31)$$

$$V_\infty(X,0) = V_\infty(X,1) = 0 \quad (32)$$

provided that the other parameters are fixed and finite.

The y-derivatives of the velocity components have dropped out of the momentum and continuity equations. This means that boundary conditions equations (31)–(32) must be abandoned. This is entirely analogous to the situation for the tangential velocity component in viscous flow over a non-slip surface at high Reynolds number [29]. It means that unless the solution fortuitously matches these conditions (without their being enforced) there will be regions of non-uniformity for the velocity solution in the neighbourhood of $Y = 0$ and $Y = 1$. This will in turn affect the species and temperature distributions through the advection effect of the horizontal component of velocity.

The equation set above, minus equations (31)–(32), will apply for the mid-section of the cavity, sufficiently far away from the horizontal surfaces. The precise meaning of this statement must await the solution of the ceiling and floor problems.

4. THE FULLY DEVELOPED SOLUTION

Being linear and only weakly coupled, these equations are easily solved to give:

$$U_\infty = 1 \quad (33)$$

$$V_\infty = \frac{Sc}{1+N}[c_1(\sigma - \theta_\infty) + c_2(\sigma - \phi_\infty) + c_3(\sigma - X)] \quad (34)$$

$$\frac{dP_\infty}{dY} = \frac{\bar{\theta}_\infty + N\bar{\phi}_\infty}{1+N} + c_4 \quad (35)$$

$$\phi_\infty = \frac{(B+1)^X - 1}{B} \tag{36}$$

$$\theta_\infty = \frac{(B+1)^{X/Le_A} - 1}{(B+1)^{1/Le_A} - 1} \tag{37}$$

where (c_4 is indeterminate, as discussed in Section 4.1)

$$c_1 = \frac{Le_A^2}{\ln^2(B+1)(Sc - Le_A)} \tag{38}$$

$$c_2 = \frac{N}{\ln^2(B+1)(Sc-1)} \tag{39}$$

$$c_3 = \frac{1}{\ln(B+1)}\left[\frac{Le_A + N}{\ln(B+1)} + c_4(1+N)\right] \tag{40}$$

$$\sigma = \frac{(B+1)^{X/Sc} - 1}{(B+1)^{1/Sc} - 1} \tag{41}$$

and the quantities with overbars are averages across the cavity, given by

$$\bar{\phi}_\infty \equiv \int_0^1 \phi_\infty \, \mathrm{d}X = \frac{1}{\ln(B+1)} - \frac{1}{B} \tag{42}$$

$$\bar{\theta}_\infty \equiv \int_0^1 \theta_\infty \, \mathrm{d}X = \frac{Le_A}{\ln(B+1)} - \frac{1}{(B+1)^{1/Le_A} - 1} \tag{43}$$

As in the heat transfer only solution of Aung [10], the temperature (and here also mass fraction) distribution is unaffected by the vertical velocity. With the substitution of mole fractions for mass fractions (arising because of the assumption of constant density rather than constant total molar concentration), the distributions, equations (36)–(37), are identical to those given by Bird, Stewart and Lightfoot [30] for simultaneous heat and mass transfer across a stagnant film of noncondensable gas; that concentration distribution being Stefan's law [31, 32].

A temperature distribution with the same form as equation (37) was obtained by Ranganathan and Viskanta [33] for a rectangular cavity in the special cases $N = -1$ and $Le_A = 1$, for which net buoyancy effects vanish everywhere, and for $N = -1$ and $B \to \infty$, for which the mass transfer induced horizontal velocities are assumed to overwhelm the buoyancy induced vertical velocities. Their temperature distribution contains a Lewis number based on the mixture specific heat, rather than the vapour specific heat. This difference arises from their neglect of species interdiffusion in the energy equation and so vanishes if the gas and vapour specific heats are equal. Reference [33] gives plots of the concentration profile for various values of B. The temperature profile is identical for $Le_A = 1$. Larger (smaller) vapour Lewis numbers reduce (increase) the departure from a linear profile.

The velocity profile here is a new result, differing from the heat transfer cubic profile because of the horizontal advection of vertical momentum due to the vapour migration and the curved buoyancy distribution; the temperature and mass fraction varying differently and nonlinearly across the cavity.

The solution has a removable singularity at $B = 0$. It is made analytic for $-1 < B < \infty$ by putting

$$V_\infty = \frac{X(X-1)}{2}\left(\frac{1-2X}{6} + c_4\right) \tag{44}$$

$$\frac{\mathrm{d}P_\infty}{\mathrm{d}Y} = \frac{1}{2} + c_4 \tag{45}$$

$$\phi_\infty = X \tag{46}$$

$$\theta_\infty = X \tag{47}$$

for $B = 0$, which is equivalent to the solution of Nelson and Woods [13].

There are also removable singularities at $Sc = 1$ and Le_A which are less important. The nondimensionalisation breaks down in the vicinity of $N = -1$, where V_∞ must be unbounded if v is nonzero.

The effect of the mass transfer driving force, B, on the buoyancy-induced vertical velocity profile is illustrated in Fig. 2.

4.1. *The vertical pressure gradient*

A more general solution that still satisfies equations (21)–(25) and boundary conditions (26)–(30) is one where c_4 varies with Y, but this must be rejected on physical grounds, as it leads to a net vertical mass flux that varies with height. This violates the conservation of mass, as by equations (18) and (33) there is no net addition of mass to the cavity through the side walls at any horizontal section for which this fully developed solution applies.

The value of the constant c_4 remains indeterminate. It is clearly related to the net vertical mass flux, which is proportional to the integral of V_∞ across the cavity. This integral depends on c_4 through c_3.

In the small mass transfer limit, the integral from $X = 0$ to 1 of V_∞ equation (44) is $-c_4/12$. For the cavity, the net vertical mass flux in this limiting case must be zero (as there is no mass flux at the walls) so that $c_4 = 0$. For an open channel, c_4 is determined by the pressure boundary conditions at the inlet and outlet.

No such simple treatment is possible for a cavity with $B \neq 0$. An inspection of Fig. 2, for which $c_4 = 0$ in all the plotted profiles, reveals that the condition of the pressure gradient balancing the mean density perturbation; i.e. $c_4 = 0$, from equation (35); does not imply a net zero vertical mass flux. Further, if $Gr^* \neq 0$, a recirculating flow would be expected, which would certainly cause the net mass added to the cavity

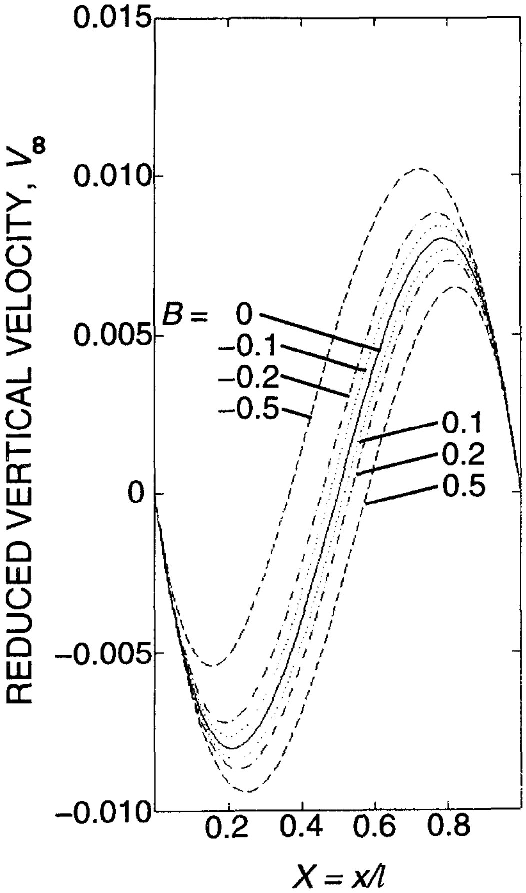

Fig. 2. Vertical, buoyancy-induced velocity profiles with finite horizontal mass transfer. The other parameters are held constant at $N = 1$, $Le_A = 0.5$, $Sc = 0.6$ and $c_4 = 0$.

through the side walls to be different in the top and bottom end regions. This difference cannot be determined without the solutions valid for these regions.

4.2. *Mass and energy fluxes at the vertical walls*

In the first approximation for the tall cavity, sufficiently far from the horizontal surfaces, the dimensionless mass transfer rate, $U(0, Y)$ or $U(1, Y)$, is unity at every point along either vertical wall, regardless of the values of the other parameters of the problem. The Nusselt number is also a constant (independent of position on the wall, and the same for each wall) but does depend on B, Le_A and H. It is given by equations (19)–(20), (33), (37) and (47) as

$$Nu^*_\infty = \begin{cases} \ln(B+1)\left\{\dfrac{1}{Le_A[(B+1)^{1/Le_A}-1]} - H\right\} & B \neq 0 \\ 1 & B = 0 \end{cases} \tag{48}$$

If the vapour condenses at one of the walls, the vapour base enthalpy, h_{A_0}, is naturally taken as the enthalpy of the vapour at T_0 relative to that of the condensed state at the same temperature. This can easily make $H \ln(B+1)$ (the dimensionless 'latent heat transfer') the dominant component of the overall energy transfer equation (48). This is the same conclusion as reached by Yan *et al.* [16], from their numerical studies of the finite length channel. For points well up the channel from the entrance, their results are well described by the above formula.

For example, in their Case II ($T_l = 50°C$, $T_0 = 30°C$, with both walls saturated with water vapour and the total pressure about 1 atm), our dimensionless parameters take on the values $H = 139$, $Le_A = 0.47$ and $B = -0.054$. Equation (48) gives $Nu^*_\infty = 8.851$. For a point 560 channel widths up from the entrance, ref. [16] gives $Nu^* = 8.849$, which is excellent agreement. With saturated boundary conditions such as these, the vapour will certainly be supersaturated in the channel, so that gas-phase condensation is possible, but this was neglected in the cited work. Equation (48) is therefore applicable.

It is obvious from this example that the humidity difference has a large effect on the energy transfer, as will often be the case.

5. FUTURE WORK

5.1. *The ceiling and floor regions*

In the solution for the flow near a non-slip surface at high Reynolds numbers, the next stage after finding the basic inviscid flow is to expand the coordinate perpendicular to the surface where the boundary condition was dropped. Here the coordinate is Y and the obvious choice for the expansion factor is $\mathscr{A}$, so that the full equations have to be solved in these regions. This will be the subject of a subsequent paper. Once solutions are found for these regions, a composite solution can be constructed for a cavity with aspect ratio arbitrary above a minimum determined by the size of the end regions.

5.2. *Stability*

It is well known [5, 34] that the corresponding heat transfer only flow (i.e. that reported by Jones and Furry [2]) is susceptible to both stationary and travelling instabilities. Thus, while the studies projected in Section 5.1 will define the aspect ratios for which the present solution is valid, a stability analysis must be performed to determine the applicable range of Gr^* and the other parameters.

6. CONCLUSION

In this paper we have presented the fully developed profiles of temperature, mass fraction and velocity, and the consequent mass and energy fluxes at the vertical walls, in the space between two parallel walls at different but constant conditions for a perfect Boussinesq mixture of a gas and a vapour with constant properties except for the mixture specific heat. The finite mass transfer effects of interfacial velocity and the interdiffusion of enthalpy are included.

The governing equations were obtained from those for a rectangular cavity by taking the limit as the aspect ratio tended to infinity. In this process, the vertical gradients of all dependent variables except pressure dropped out of the field equations. As a consequence, the velocity boundary conditions at the horizontal surfaces had to be abandoned. The solutions presented thus apply equally well to flow in open channels as well as cavities, but only sufficiently far from the ends. The question of how far is sufficiently far must await the solution for the flow in the end regions, which will be treated in a subsequent paper.

In the zero mass transfer limit, the temperature and mass fraction vary linearly across the cavity while the horizontal velocity is zero and the vertical velocity is described by a cubic (odd symmetric for a cavity, and tending toward an even symmetric parabola as the imposed pressure gradient becomes large compared to the density perturbation for an open channel). For finite mass transfer, the horizontal velocity is nonzero, but must be constant. This horizontal advection profoundly alters the transport of vapour, energy and momentum across the cavity, so that while their profiles can still be expressed in closed form, this involves exponential functions rather than polynomials. The expression derived for the wall energy flux shows that the overall cavity energy transfer can be very much higher if a vapour condenses at the walls.

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